The Point-Matching Solution for Magnetically Tunable Cylindrical Cavities and Ferrite Planar Resonators

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***Abstract*—This paper presents an exact field theory treatment for a cylindrical cavity containing a full-height triangular ferrite post as well as for ferrite planar resonators of arbitrary shape. The knowledge of the resonant frequencies of the cavity is essential for the construction of circulators with a triangular ferrite post; those of the planar circuits are needed for the design of microwave integrated circuits. The treatment is general and depends neither on the location of the ferrite post inside the cavity nor on the geometry of the planar resonator. The solution of the wave equations in the ferrite material and in a possible surrounding air region is written as an infinite summation of cylindrical modes. In the case of the cavity, the individual modes are exactly matched along the internal cylindrical metallic boundary of the cavity. The fields at the ferrite–air interface in both cases are matched using the point-matching technique, which leads to a finite system of homogeneous, simultaneous equations for which the determinantal equation must be zero. An example of a cavity with a triangular ferrite post is studied and calculated, and measured results are compared. Furthermore, examples of application of the theory on triangular and quadratic planar resonators are described and compared with published experimental measurements.**

I. Introduction

THE PRACTICAL application of magnetically tunable cavities, especially in circulators, has generated interest in the ferrite-filled cavity resonator problem, partially ferrite-filled cavity resonator problem, and the planar ferrite circuit problem.

A number of publications on ferrite-filled cavities is known, of which only some can be mentioned here. Kales and Gamo [4], [5] already very early showed that z-dependent pure TE- or TM-modes cannot exist in partially ferrite-filled cylindrical geometries. Heller and Bussey [1]–[3] have pointed out that TM_{lm0} modes exist in the cylindrical cavity containing a longitudinally magnetized ferrite rod and that the problem is tractable if the rod is of a circular cross section. TM_{lm0} modes are modes in which the electromagnetic fields are independent of the height coordinate of the cylinder, and l represents the number of the full-period variations of the field components with respect to θ, and m represents the number of half-period variations of the field components with respect to r. Ferrite-filled resonators of a rectangular cross section for the first time (to the knowledge of the authors) have been treated by Brand [6], who gave a solution for the z-independent modes of these resonators. Bolle [7] described a variational approach to calculate a ferrite-filled cavity of a rectangular cross section. A complete description of possible field solutions in ferrite-filled cavities can be found in [13].

Only a few publications on ferrite planar circuits are known. Okoshi and Miyoshi [11] introduced the concept of planar circuits for use in microwave integrated circuits, and Miyoshi, Yamaguchi, and Goto [10] described two methods based on a contour-integral solution and on a field expansion method to calculate the properties of planar circuits on a ferrite substrate. Recently, Helszajn [18] described the fields of modes in a demagnetized planar circuit of a triangular geometry, using the field solution given by Schelkunoff [12].

In this paper a method will be described which can be used to calculate microwave cylindrical cavities with a ferrite post of an arbitrary cross section as well as planar circuits of arbitrary geometry on a ferrite substrate. The method bases on the point matching of the electromagnetic fields. The application of this method and its validity have been described in fundamental papers by Lewin and Nielsen [8], [9].

As a first example for the application of the developed method, a cylindrical cavity with a full-height triangular ferrite post placed inside the cavity and a dc magnetic field applied normal to the plane of the cavity (Fig. 1) is considered. Since the triangular ferrite post is assumed to be smooth in the plane of the cavity, the scattered electric fields will be normal to the plane of the cavity, and the magnetic fields will be transversal. The individual modes of the cavity are exactly matched at the internal metallic cylindrical boundary whereas the fields of the cylindrical modes in the ferrite post are matched to those of the cylindrical modes outside the post at the ferrite–air interface using the point-matching technique. This leads to a finite system of homogeneous, simultaneous equations for which the determinantal equation must be zero. Field distributions of the lowest order modes are computed and measured resonant frequencies are compared to the calculated eigenvalues.

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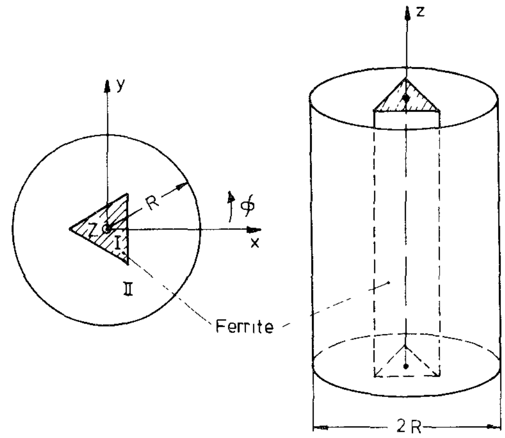


Fig. 1. Cylindrical cavity containing a longitudinally magnetized ferrite three-sided prism.

In two further examples, triangular and rectangular planar circuits on a ferrite substrate are analyzed using the same method. The fields of the planar circuits are described by cylindrical modes; they are matched to satisfy the magnetic wall conditions at the boundary of the circuits using the point-matching method. Eigenvalues and field distributions of the lowest order modes of the circuits are calculated. Published measurements of resonant frequencies corresponding to the lowest eigenvalues are compared with the calculated frequencies.

II. Theoretical Analysis

A. Cylindrical Cavity Containing a Longitudinally Magnetized Ferrite Three-Sided Prism

The cylindrical cavity may be considered as divided into two regions, the ferrite three-sided prism and the surrounding air region as shown in Fig. 1. The electric field E_z in the ferrite (region I) satisfies the homogeneous Helmholtz equation in cylindrical coordinates.

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}+k_f^2\right]\cdot E_z=0 \tag{1}$$

where

$$k_f^2=\omega^2\mu_0\epsilon_0\,\mu_{\mathrm{eff}\,1}\epsilon_{fr}$$

$$\mu_{\mathrm{eff}\,1}=\frac{\mu^2-k^2}{\mu}$$

μ and k are the diagonal and off-diagonal permeability tensor elements in the xy plane, and ϵ_{fr} is the relative permittivity of the ferrite material.

The solution of (1) takes the following form:

$$E_z(r,\phi)=\begin{cases}\displaystyle\sum_{n=-\infty}^{\infty}a_nJ_n(k_fr)e^{-jn\phi}, & \mu_{\mathrm{eff}\,1}>0\\ \displaystyle\sum_{n=-\infty}^{\infty}a_nI_n(k_fr)e^{-jn\phi}, & \mu_{\mathrm{eff}\,1}<0.\end{cases} \tag{2}$$

From the Maxwell's equations, the azimuthal and radial magnetic-field components, when $\mu_{\mathrm{eff}\,1}>0$, are

$$H_\phi(r,\phi)=-jY_f\sum_{n=-\infty}^{\infty}a_n\left\{J_n'(k_fr)+\frac{k}{\mu}\frac{n}{k_fr}J_n(k_fr)\right\}e^{-jn\phi} \tag{3}$$

$$H_r(r,\phi)=Y_f\sum_{n=-\infty}^{\infty}a_n\left\{\frac{k}{\mu}J_n'(k_fr)+\frac{n}{k_fr}J_n(k_fr)\right\}e^{-jn\phi}. \tag{4}$$

Similarly, fields outside the ferrite (region II) are of the form

$$E_z(r,\phi)=\sum_{n=-\infty}^{\infty}\{b_nJ_n(k_0r)+c_nY_n(k_0r)\}e^{-jn\phi} \tag{5}$$

$$H_\phi(r,\phi)=-jY_0\sum_{n=-\infty}^{\infty}\{b_nJ_n'(k_0r)+c_nY_n'(k_0r)\}e^{-jn\phi} \tag{6}$$

$$H_r(r,\phi)=Y_0\sum_{n=-\infty}^{\infty}\left\{b_n\frac{n}{k_0r}J_n(k_0r)+c_n\frac{n}{k_0r}Y_n(k_0r)\right\}e^{-jn\phi} \tag{7}$$

where

$$Y_f=\sqrt{\frac{\epsilon_0\epsilon_{fr}}{\mu_0\mu_{\mathrm{eff}\,1}}},\qquad Y_0=\sqrt{\frac{\epsilon_0}{\mu_0}}$$

and

$$k_0^2=\omega^2\mu_0\epsilon_0.$$

The prime denotes differentiation with respect to the argument, and a_n, b_n, and c_n are the unknown field amplitudes which can be obtained from the continuity conditions of the tangential field components at the ferrite–air interface and at $r=R$.

The boundary conditions for the E_z component at $r=R$ are

$$E_z(R,\phi)|_{\mathrm{air}}=0,\qquad 0\leqslant\phi\leqslant 2\pi. \tag{8}$$

From (5) we get

$$c_n=-\frac{J_n(k_0R)}{Y_n(k_0R)}b_n. \tag{9}$$

On the boundary between the air (surrounding the ferrite) and the ferrite, the fields must be matched. The boundary depends on the geometry of the ferrite, and only the ferrite three-sided prism is considered here.

The continuity conditions for the electric field component $E_z(r,\phi)$ and the tangential magnetic field component $H_{\mathrm{tan.}}(r,\phi)$ can be applied at any point m ($r=r_m$) along the ferrite–air interface using (2)–(7) and (9) to give

$$\sum_{n=-\infty}^{\infty}a_nJ_n(k_fr_m)e^{-jn\phi_m}$$
$$+\sum_{n=-\infty}^{\infty}b_n\left\{Y_n(k_0r_m)\frac{J_n(k_0R)}{Y_n(k_0R)}-J_n(k_0r_m)\right\}e^{-jn\phi_m}=0 \tag{10}$$

for the E_z component and

$$\sum_{n=-\infty}^{\infty} a_n \cdot Y_f \cdot \left[\sin(\phi_{1m}) \cdot \left\{ \frac{k}{\mu} J_n'(k_f r_m) + \frac{n}{k_f r_m} J_n(k_f r_m) \right\} - j\cos(\phi_{1m}) \left\{ J_n'(k_f r_m) + \frac{k}{\mu} \frac{n}{k_f r_m} J_n(k_f r_m) \right\} \right] \cdot e^{-jn\phi_m}$$
$$+ \sum_{n=-\infty}^{\infty} b_n \cdot Y_0 \cdot \left[\sin(\phi_{1m}) \cdot \frac{n}{k_0 r_m} \left\{ Y_n(k_0 r_m) \frac{J_n(k_0R)}{Y_n(k_0R)} - J_n(k_0 r_m) \right\} - j\cos(\phi_{1m}) \cdot \left\{ Y_n'(k_0 r_m) \frac{J_n(k_0R)}{Y_n(k_0R)} - J_n'(k_0 r_m) \right\} \right] \cdot e^{-jn\phi_m} = 0 \quad (11)$$

for the tangential magnetic field. ϕ_{1m} is the angle between the side of the ferrite triangle and $H_\phi(r_m, \phi_m)$. ϕ_{1m} is positive when it takes the same direction as ϕ_m and negative, otherwise (Fig. 2).

The truncation of the infinite series in the equation systems (10) and (11) amounts to taking into account only a finite number of cylindrical modes, on the assumption that the neglected modes have much smaller amplitudes.

Consider N cylindrical modes and Q matching points at each side of the triangular ferrite post such that

$$2N+1=3Q. \quad (12)$$

Equations (10) and (11) then can be written in the form of quadratic matrices:

$$A \cdot a + B \cdot b = 0 \quad (13)$$
$$A1 \cdot a + B1 \cdot b = 0 \quad (14)$$

where

$$A=[A_{li}], \qquad A_{li} = J_i(k_f r_l) e^{-ji\phi_l}$$

$$B=[B_{li}], \qquad B_{li} = \left[Y_i(k_0 r_l) \frac{J_i(k_0R)}{Y_i(k_0R)} - J_i(k_0 r_l) \right] \cdot e^{-ji\phi_l}$$

$$A1=[A1_{li}], \qquad A1_{li} = Y_f \left[\sin(\phi_{1l}) \cdot \left\{ \frac{k}{\mu} J_i'(k_f r_l) + \frac{i}{k_f r_l} J_i(k_f r_l) \right\} - j\cos(\phi_{1l}) \cdot \left\{ J_i'(k_f r_l) + \frac{k}{\mu} \frac{i}{k_f r_l} \cdot J_i(k_f r_l) \right\} \right] \cdot e^{-ji\phi_l}$$

$$B1=[B1_{li}], \qquad B1_{li} = Y_0 \cdot \left[\sin(\phi_{1l}) \cdot \frac{i}{k_0 r_l} \cdot \left\{ Y_i(k_0 r_l) \cdot \frac{J_i(k_0R)}{Y_i(k_0R)} - J_i(k_0 r_l) \right\} - j\cos(\phi_{1l}) \cdot \left\{ Y_i'(k_0 r_l) \frac{J_i(k_0R)}{Y_i(k_0R)} - J_i'(k_0 r_l) \right\} \right] \cdot e^{-ji\phi_l}$$

$$a = [\, a_i \,] \quad b = [\, b_i \,]$$

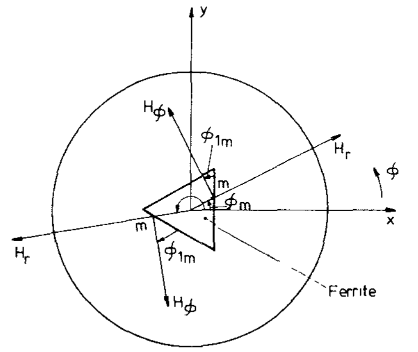


Fig. 2. Schematic representation of the cylindrical cavity with a triangular ferrite post and symbols used in the point-matching equations.

with $i=-N, -N+1, \cdots, N-1, N$ and $l=1,2,3,\cdots,3Q$.

Equations (13) and (14) can be combined as

$$Z \cdot b = 0 \quad (15)$$

where the matrix Z is given by

$$Z = A^{-1} \cdot B - A1^{-1} \cdot B1. \quad (16)$$

From the nontrivial condition of (15), we have

$$\det(Z) = 0. \quad (17)$$

The matrix Z is of the order $(2N+1)$, with N being the number of cylindrical modes taken into account. Equation (17) gives the resonant frequencies of the cavity.

B. Ferrite Planar Resonators

A ferrite planar circuit is an electrical circuit whose thickness in one direction is much less than one wavelength and whose dimension in the orthogonal directions is comparable to the wavelength [11]. It consists of an arbitrarily shaped thin conductor on a ferrite substrate which is fully metallized on its backside with a dc magnetic field perpendicular to the substrate plane. The thickness of the planar circuit is d. Because the spacing d is much smaller than the wavelength and since the ferrite substrate is assumed to be homogeneous and linear, only the field components E_z, H_ϕ, and H_r with no variation along the dc bias field direction are considered. (It is assumed that the periphery of the ferrite planar circuit is open circuited where the coupling ports are absent.) In other words, the tangential magnetic field is assumed to be zero over the whole periphery in the case of the ferrite planar resonators. A simple correction for the fringing magnetic field effect is to enlarge the periphery outwards by the amount of $0.447\, d \cdot k$ $(k=0.2)$ [10] in advance of the analysis. If a correction of higher accuracy is wanted, the method presented in [16] can be used.

For the z-independent waves in this structure, as in the case of the cylindrical cavity containing a ferrite post, the electric field E_z and the magnetic fields H_ϕ and H_r take the same form as in (2)–(4), respectively.

The boundary condition of the tangential magnetic field at the periphery can be applied at any point m $(r=r_m,\ \phi=\phi_m)$ along the periphery of the ferrite planar

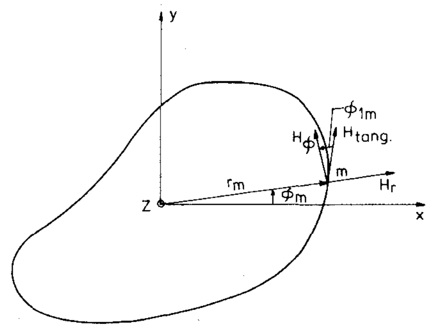


Fig. 3. Schematic representation of a ferrite planar resonator and symbols used in the point-matching equations.

resonator ($\mu_{\text{eff }1}>0$, Fig. 3) to give

$$\sum_{n=-\infty}^{\infty} a_n\cdot Y_f\Bigg[\sin(\phi_{1m})\cdot\left\{\frac{k}{\mu}J_n'(k_fr_m)+\frac{n}{k_fr_m}J_n(k_fr_m)\right\}$$
$$-j\cos(\phi_{1m})\cdot\left\{J_n'(k_fr_m)+\frac{k}{\mu}\frac{n}{k_fr_m}J_n(k_fr_m)\right\}\Bigg]\cdot e^{-jn\phi_m}=0. \tag{18}$$

Consider N cylindrical modes and Q matching points at the periphery of the resonator such that

$$2N+1=Q. \tag{19}$$

Equation (18) then can be written in the form of a quadratic matrix equation

$$C\cdot a=0 \tag{20}$$

where

$$C=[C_{li}],\qquad C_{li}=Y_f\Bigg[\sin(\phi_{1l})\cdot\left\{\frac{k}{\mu}J_i'(k_fr_l)+\frac{i}{k_fr_l}J_i(k_fr_l)\right\}$$
$$-j\cos(\phi_{1l})\cdot\left\{J_i'(k_fr_l)+\frac{k}{\mu}\frac{i}{k_fr_l}J_i(k_fr_l)\right\}\Bigg]\cdot e^{-ji\phi_l}$$

with $i=-N,-N+1,\cdots,N-1,N$, and $l=1,2,3,\cdots,Q$. From the nontrivial condition of (20), we have

$$\det C=0. \tag{21}$$

This equation gives the resonant frequencies of the resonator.

Principally, other than the cylindrical functions, e.g., a complete function system in rectangular or triangular coordinate systems [12] can be used to find a resonance condition equivalent to (20). The cylindrical expansion functions are chosen here because of the simple formulation of the field equations in the ferrite material in a cylindrical coordinate system.

III. Testing of the Analysis

In the following example, the ferrimagnetic material TT1-109 is assumed to be lossless. As an example of the computer analysis described so far, the resonant frequencies of a cylindrical cavity with a cylindrical ferrite rod ($R=1.85$ cm, $r_i=0.3$ cm) were computed first to check the computation accuracy.

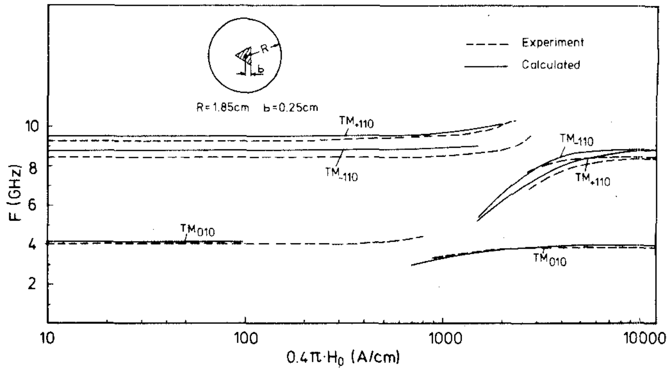


Fig. 4. Magnetic tuning characteristics of a cylindrical cavity with a triangular ferrite post.

It should be noted that det Z has been found to be predominantly real or imaginary depending on the number of cylindrical modes taken into account, and that a change of the sign of det Z does not always lead to a nontrivial condition of (15); it may lead to a singularity. A Newton–Raphson method has been employed to select the nontrivial conditions of (15), i.e., det $Z=0$.

The case of 15 matching points on the ferrite–air interface are considered. Accordingly, $N=7$ cylindrical modes are retained. Two cases are considered, below the ferrimagnetic resonance ($H_0=80$ A/cm) and above the ferrimagnetic resonance ($H_0=4000$ A/cm). By comparing the calculated eigenvalues for these two cases ($H_0=80$ A/cm, $H_0=4000$ A/cm) with the theoretical ones, which should be given by the roots of

$$\sqrt{\frac{\mu_{\text{eff }1}}{\epsilon_r}}\,J_n(k_rr_i)\cdot\left\{J_n'(k_0r_i)-\frac{J_n(k_0R)}{Y_n(k_0R)}Y_n'(k_0r_i)\right\}$$
$$-\left\{J_n'(k_fr_i)+\frac{k}{\mu}\frac{n}{k_fr_i}J_n(k_fr_i)\right\}\left\{J_n(k_0r_i)-\frac{J_n(k_0R)}{Y_n(k_0R)}\right.$$
$$\left.\cdot Y_n(k_0r_i)\right\}=0,\qquad n=0,\pm1,\pm2,\cdots \tag{22}$$

it is found that the computation error was within 1.0 percent for the two cases taking into account 7 cylindrical modes.

IV. Numerical Results and Discussions

A. Cylindrical Cavity Containing an Axially Magnetized Ferrite Three-Sided Prism

In this example, 11 matching points on each side of the triangular ferrite are considered. Accordingly, 17 cylindrical modes ($N=16$) at the ferrite–air interface are retained. The characteristics of cylindrical cavities with a three-sided prism ferrite, of which nothing has been reported, were studied. Fig. 4 shows the magnetic tuning characteristics of the cavity. The calculated resonant frequencies shown in solid curves are found to be in fairly good agreement with the measured values, below as well as above the ferrimagnetic resonance.

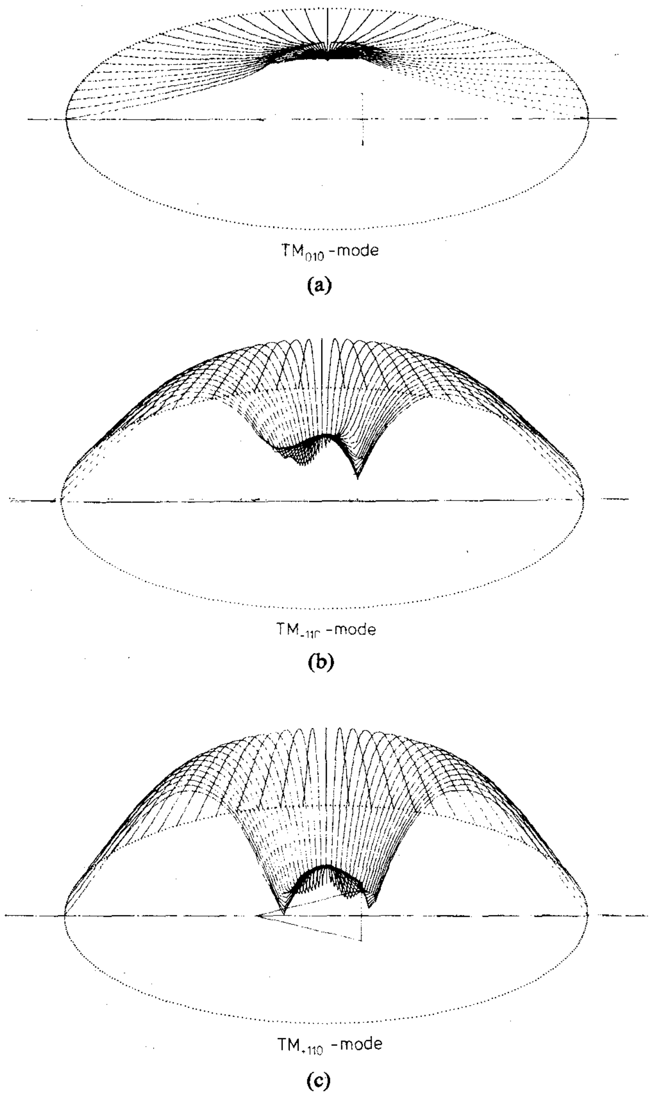


Fig. 5. Relative electric field intensities E_z across the cylindrical cavity with a triangular ferrite post. (a) $H_0=80$ A/cm, $F=4.127$ GHz. (b) $H_0=160$ A/cm, $F=8.819$ GHz. (c) $H_0=160$ A/cm, $F=9.524$ GHz.

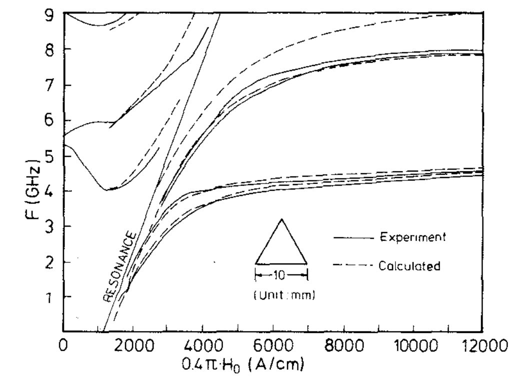


Fig. 6. Magnetic tuning characteristics of a triangular ferrite planar resonator; —— measured values given by [10]; ---- values computed from this theory.

In the experiment, a longitudinally equilateral triangular ferrite post with a radius of the inscribed circle of 0.25 cm of the ferrimagnetic material TT1-109 was centered in a cylindrical cavity of the radius $R=1.85$ cm. Fig. 5 (a), (b), and (c) shows the computed instantaneous relative

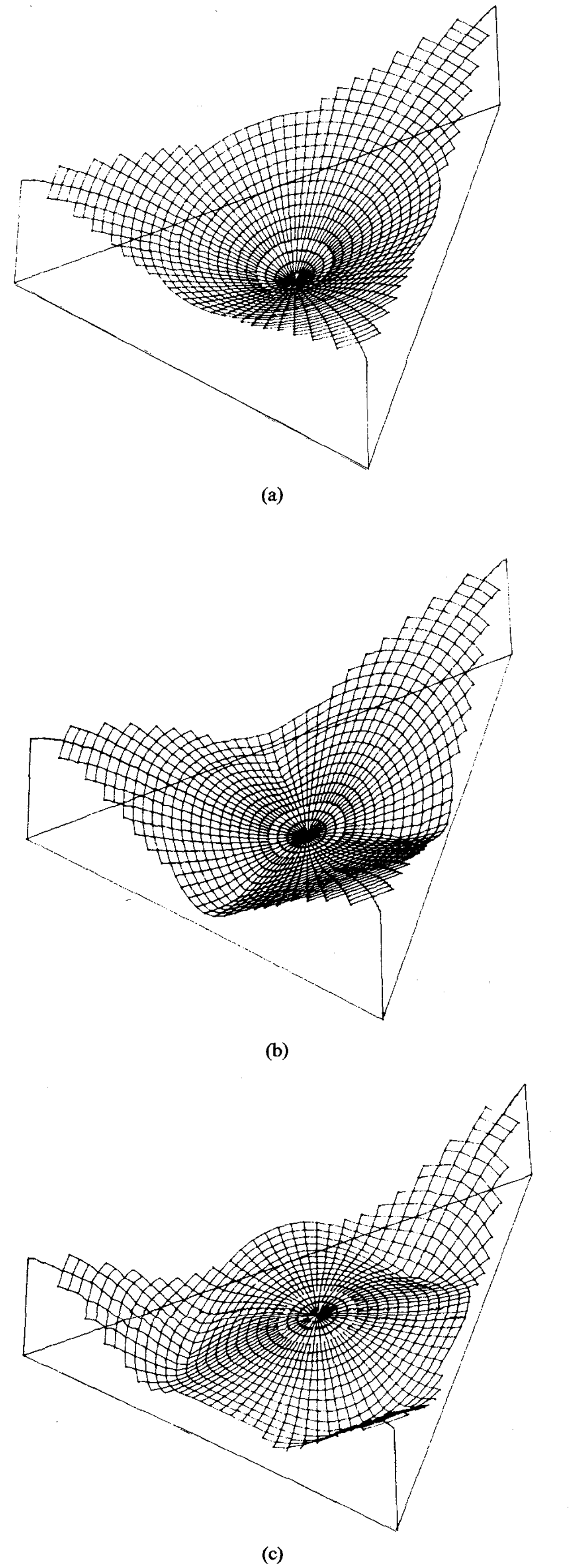


Fig. 7. Relative electric field intensities of a triangular ferrite planar resonator. (a) $H_0=1035$ A/cm, $F=4.02$ GHz. (b) $H_0=1035$ A/cm, $F=5.85$ GHz. (c) $H_0=1035$ A/cm, $F=8.52$ GHz.

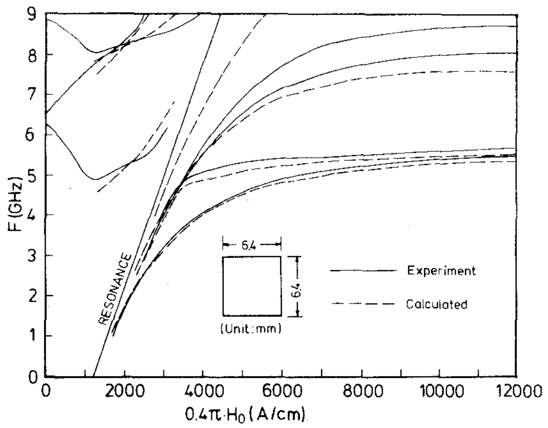


Fig. 8. Magnetic tuning characteristics of a square ferrite planar resonator; —— measured values given by [10]; ---- values computed from this theory.

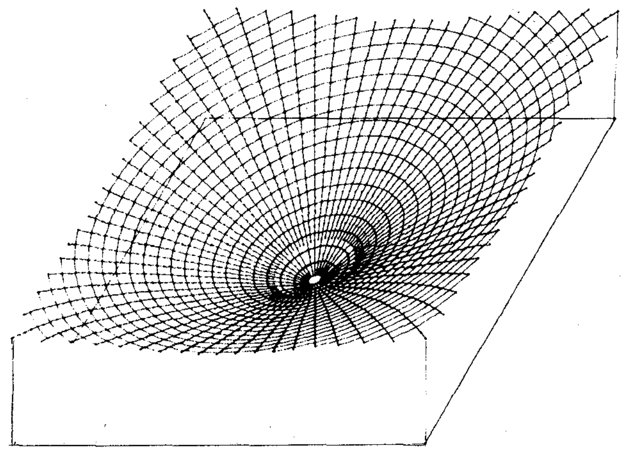
electric field intensities $|E_z|$ across the cavity for three resonant frequencies. These figures indicate that the electric field attains almost a null value at the internal metallic boundary of the cavity as was required in this case.

For the lowest resonant frequency, the electric field attains a maximum value at the center of the cavity and behaves more or less like the Bessel function of zero order. It corresponds to the lowest order mode TM_{010}. For the first two higher resonant frequencies, the electric field at the center of the cavity does not indicate a null as in the case of the $TM_{\pm 110}$ mode of the cylindrical ferrite post [17]. This means that the zeroth-order space harmonic still has a relatively large amplitude value, and consequently, it affects these resonances. According to the field distributions, it is easy to classify the field modes of the three resonant frequencies similarly to the case of the cavity with a cylindrical ferrite post. The resonant frequencies in the case of triangular ferrite posts are realized with the interaction of many space harmonics, whereas any resonant frequency in the case of a cylindrical ferrite post can be described in terms of the corresponding rotating space harmonic.

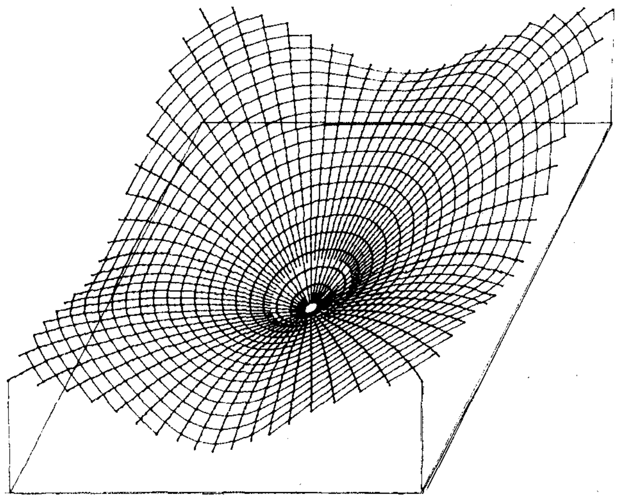
B. Ferrite Planar Resonators

Point-matching techniques are used to study the characteristics of triangular and square ferrite planar resonators. The case of 15 matching points on the periphery of each resonator is considered. Accordingly, 8 cylindrical modes are retained. Fig. 6 shows the magnetic tuning characteristics of the triangular ferrite planar resonator with the same physical dimensions and using the same ferrimagnetic material as given by [10] (for comparison with their published experimental measurements). The calculated values, based on the technique adopted in this paper, are found to be in quite good agreement with the published experimental measurement, below as well as above the ferrimagnetic resonance.

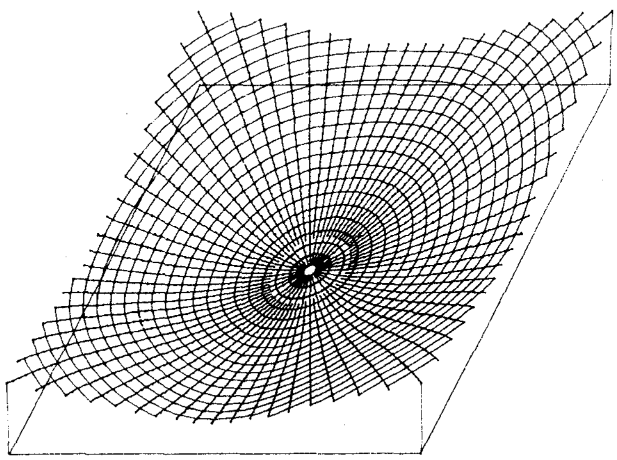
Fig. 7 (a), (b), and (c) show the computed instantaneous relative electric field intensities across the resonator for three resonant frequencies for the same dc magnetic field



(a)



(b)



(c)

Fig. 9. Relative electric field intensities of a square ferrite planar resonator. (a) $H_0=1035$ A/cm, $F=4.55$ GHz. (b) $H_0=1035$ A/cm, $F=7.52$ GHz. (c) $H_0=1035$ A/cm, $F=7.82$ GHz.

($H_0=1035$ A/cm). As Fig. 7 shows, the field intensity of the third resonance does not vanish in the center of the cavity; therefore, it can be concluded that the zeroth-order space harmonic of the field expansion functions (zeroth-order Bessel function) affects the field distribution of this resonance.

In the case of the square ferrite planar resonator, the magnetic tuning characteristics and the instantaneous relative electric field intensities across the resonator are shown in Figs. 8 and 9 (a), (b), and (c), respectively. It is found from the figures that almost the same resonant characteristics as obtained for a triangular resonator result.

V. Conclusion

The analysis given can be applied to cylindrical cavities with arbitrarily shaped, longitudinally magnetized ferrite posts as well as to arbitrarily shaped, ferrite planar resonators or circuits. The numerical results obtained for all the examples are found to be in agreement with the experimental results and the previously published experimental ones. The point-matching technique will be useful in the design and analysis of ferrite planar circuits as well as magnetically tunable cavities.

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Theory of Infrared and Optical Frequency Amplification in Metal–Barrier–Metal Diodes

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***Abstract*—The near-infrared and optical frequency power gain of a metal–barrier–metal (MBM) point contact diode exhibiting a negative differential resistance region in its current–voltage characteristic is derived as a function of frequency. The diode is treated as a traveling-wave amplifier. The starting point for the analysis is the known electric and magnetic field distribution of the surface waves that propagate in the oxide barrier layer between the diode whisker and substrate, assuming no tunneling current is present. Then the differential tunneling conductance is introduced, and the electric and magnetic field distribution is used to find the propagation constant of the equivalent transmission line formed by the diode structure. It is shown that if the differential tunneling conductance is negative, gain can result. It is shown theoretically that the diode amplifier can provide approximately a 6-dB gain from the CO_2 laser frequency to the He–Ne laser frequency.**

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